

Abstract

By employing the isospectral reduction (ISR) technique, we can design and study systems with gains/loss and complex hoppings that reduce to the paradigmatic Hatano-Nelson and non-Hermitian Su-Schrieffer-Heeger (NH SSH) models. Our approach reveals that these models may exhibit energy (or frequency)-dependent non-Hermitian skin effect, where eigenstates can simultaneously localize on either ends of the systems, with different localization lengths. Moreover, we predict the existence of various topological edge states, pinned at non-zero energies, with different exponential envelopes, depending on their energy. Overall, our work sheds new light on the nature of topological phases and the non-Hermitian skin effect in one-dimensional systems. This poster shows an example of our results with the NH SSH model.

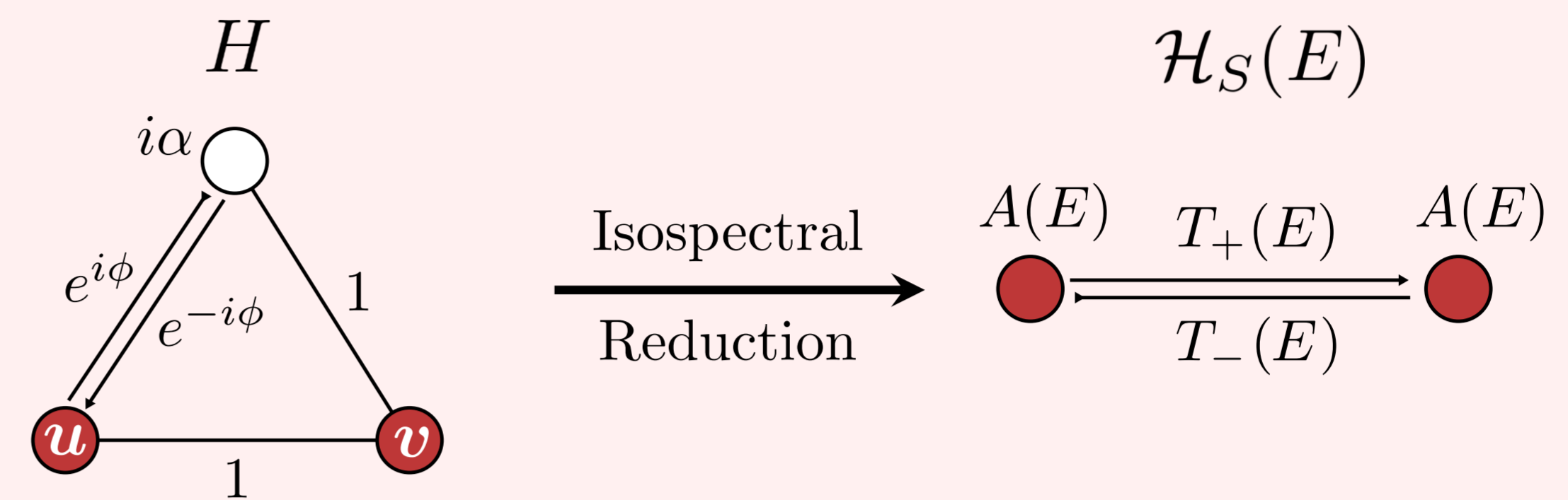
Isospectral Reduction

$$H \equiv \begin{pmatrix} H_{SS} & H_{S\bar{S}} \\ H_{\bar{S}S} & H_{\bar{S}\bar{S}} \end{pmatrix}$$

Partitioning the Hamiltonian into a set of sites S and its complement

Fig.1: Isospectral Reduction of a 3-site Hamiltonian H , with on-site loss (white) and a flux through the plaquette, to an energy-dependent two-site Hamiltonian $\mathcal{H}_S(E)$

$$H |\psi\rangle = E |\psi\rangle \Rightarrow \mathcal{H}_S(E) = H_{SS} - H_{S\bar{S}} (H_{\bar{S}\bar{S}} - E\mathbb{1})^{-1} H_{\bar{S}S}$$



$$\mathcal{H}_S(E) = \begin{pmatrix} A(E) & T_+(E) \\ T_-(E) & A(E) \end{pmatrix}$$

$$T_{\pm}(E) = v(E) \pm g(E)$$

$$A(E) = \frac{1}{E - i\alpha},$$

$$T_{\pm}(E) = 1 + \frac{e^{\pm i\phi}}{E - i\alpha}$$

Emergent NH SSH Model

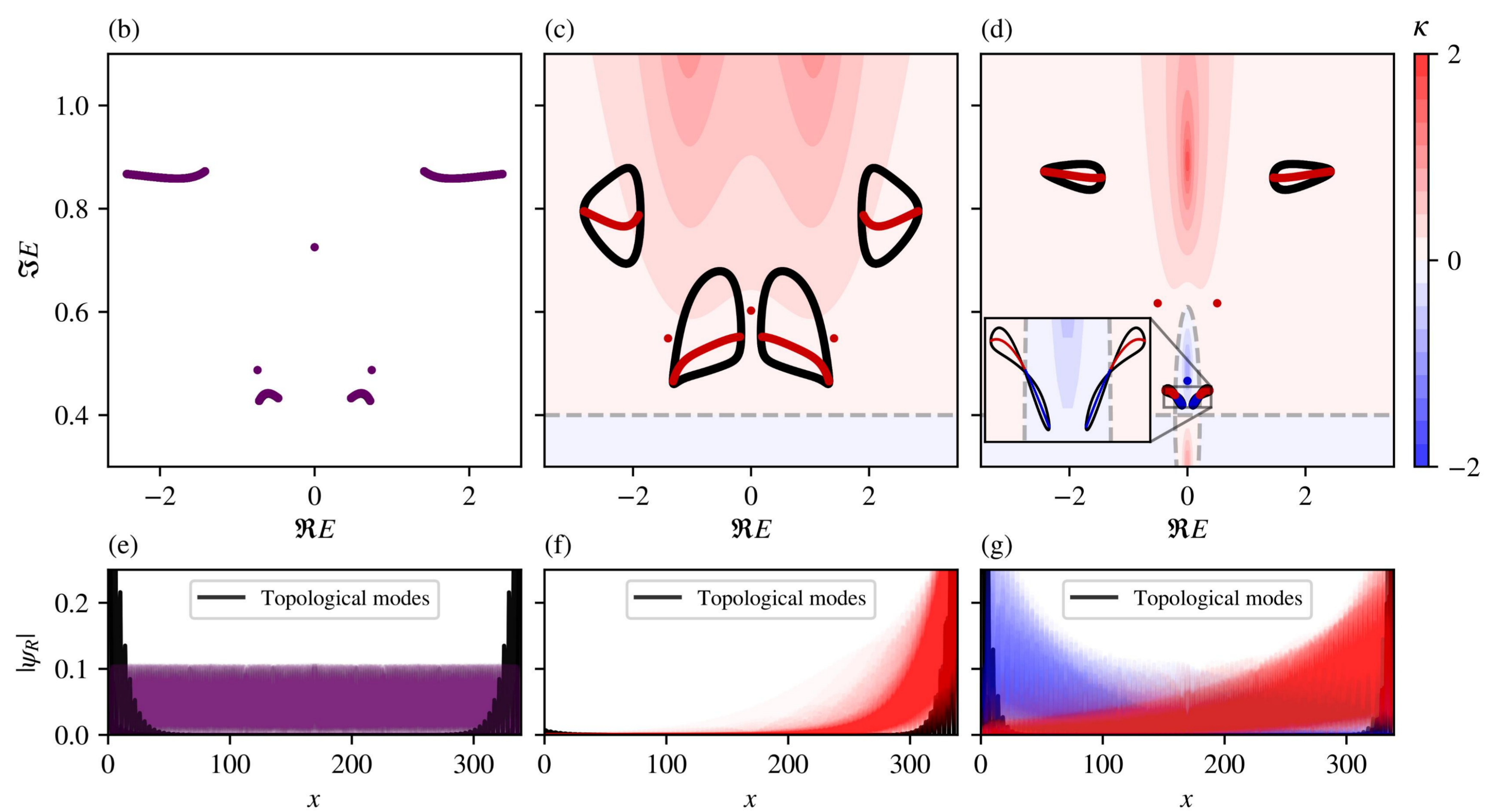
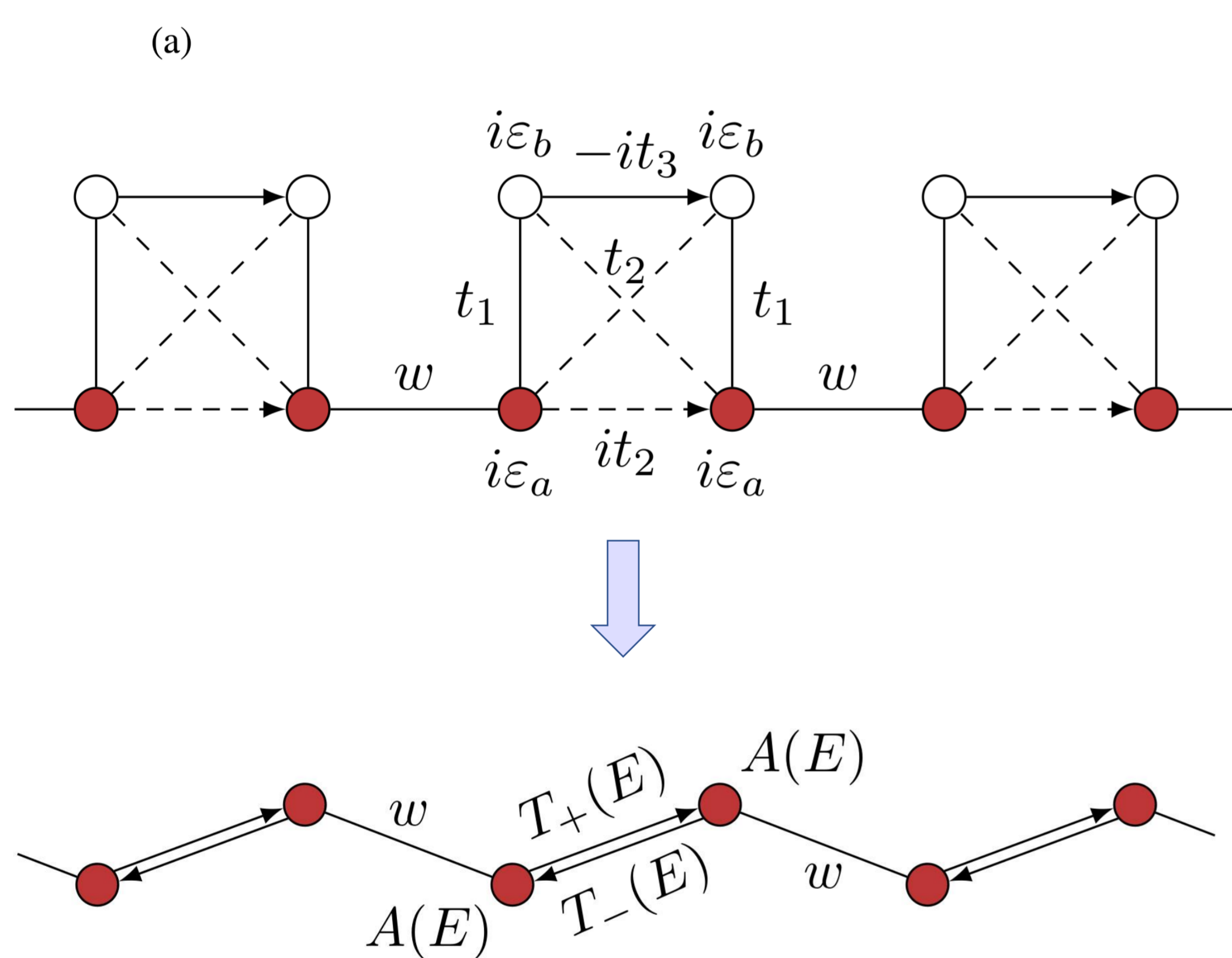
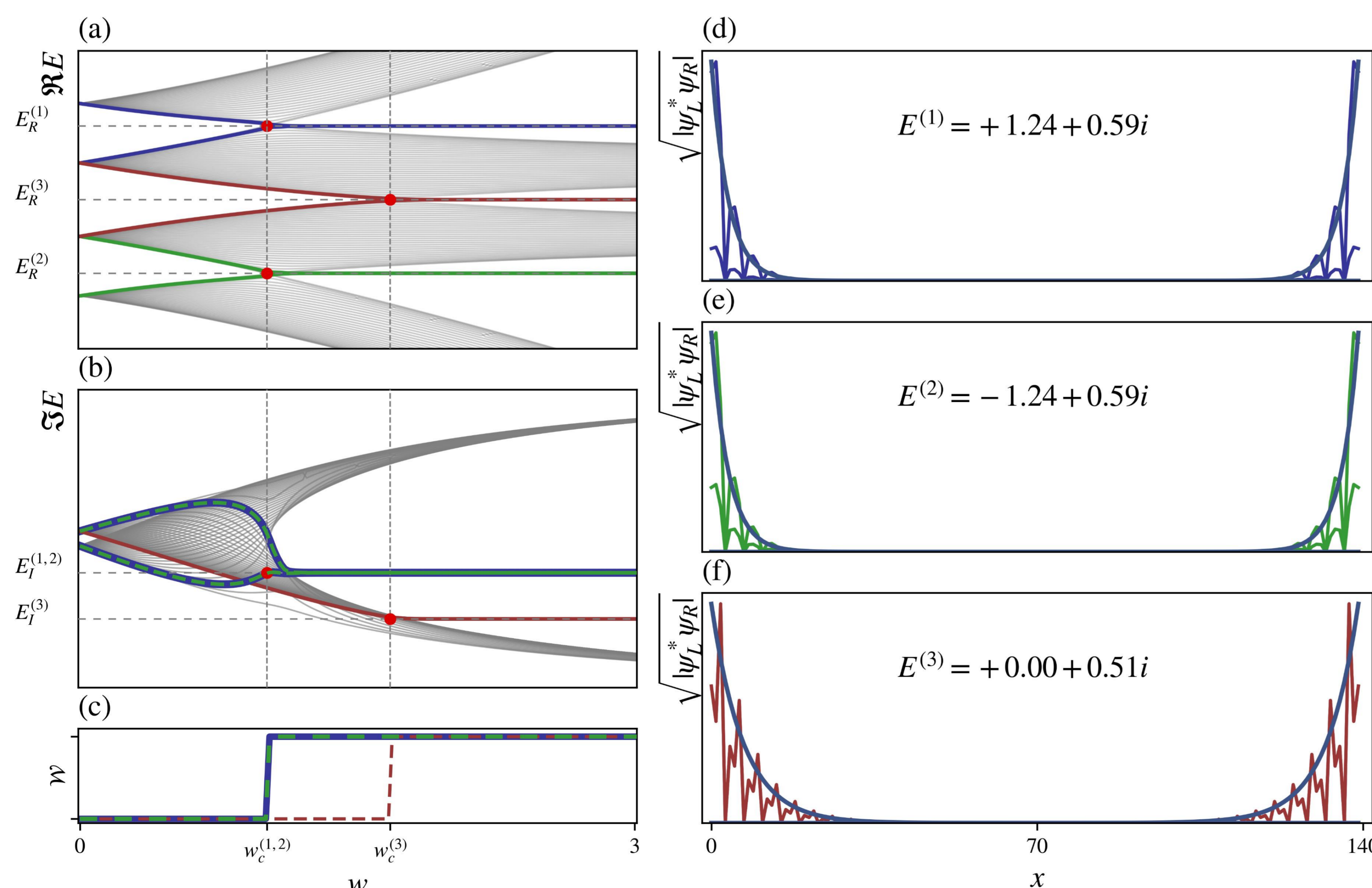


Fig.2: (a) 4-band model and its ISR to the NH SSH model. (b-d) PBC (black) and OBC (coloured) spectra. The inset of (d) shows the twisting of the PBC loop, indicating the presence of a Bloch point, and thereby the reversal of the skin direction. (e-g) Magnitude of right eigenvectors: note the presence of the (doubly-degenerate) topological edge modes (see next box for explanation).

Topological Modes of the Emergent NH SSH



Topological Characterization

$$A(E_t) - E_t = 0 : \text{ Latent sublattice symmetry}$$

$$\mathcal{W}(E_t) = \begin{cases} 0, & \text{if } |w| < \sqrt{|v^2(E_t) - g^2(E_t)|} \\ 1, & \text{if } |w| > \sqrt{|v^2(E_t) - g^2(E_t)|} \end{cases}$$

Fig.3. (a,b) OBC spectrum as a function of intercell hopping strength w . The red circles correspond to the predicted transition energies and critical w values. We see that there are three energies at which topological modes exist (see spectra of Fig.2). They are solutions of the above equation, enforcing sublattice symmetry. (c) Transition of the winding numbers for all three transition energies. There is a clear agreement with the appearance of the edge modes seen in (a,b). (d-f) Biorthogonal wavefunction amplitude of the corresponding topological modes. Each one is doubly degenerate, by virtue of the latent sublattice symmetry.

Conclusions

- We have shown how the NH SSH model can be obtained from the ISR, and we used it to predict energy-dependent skin effects and topological edge states.
- We designed lattices and a construction principle that result in emergent Hatano-Nelson and NH SSH models.
- Outlook: apply the same reductions to other paradigmatic models

References

1. Article to be uploaded on arXiv soon.
2. E. Bergholtz et al. RMP **93**, 015005 (2021)
3. L. Bunimovich and B. Webb, LAA **473**, 1429 (2012)