Coupled Fibonacci Quasicrystals

A. Moustaj, M. Röntgen, C. Morfonios, P. Schmelcher, and C. Morais Smith, *arXiv 2208.05178* (2022)

We study two identical Fibonacci chains coupled to each other in different ways. We find that this setup allows for a rich variety of effects. Depending on the coupling scheme used, the resulting system (i) possesses an eigenvalue spectrum featuring a richer hierarchical structure compared to the spectrum of a single Fibonacci chain, (ii) shows a coexistence of Bloch and critical eigenstates, or (iii) possesses a large number of degenerate eigenstates, each of which is perfectly localized on only four sites of the system. The latter realizes a perfectly flat band.

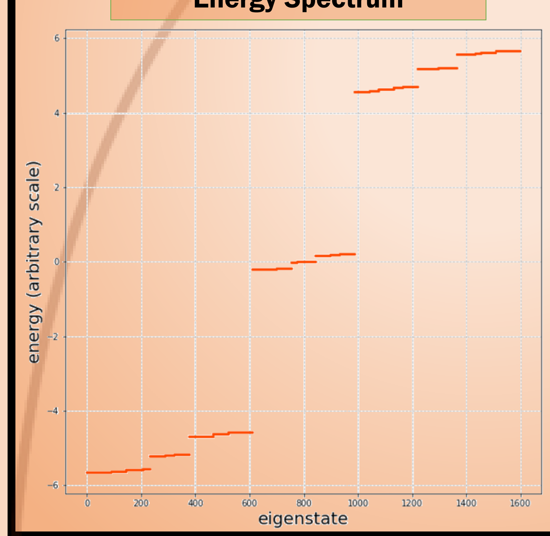
Follow the Fibonacci Spiral

Hamiltonian and block-renormalization structure

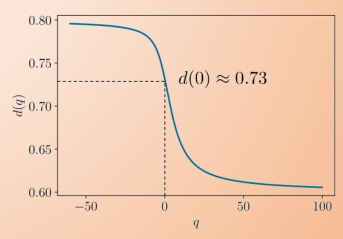
$$H = \sum_{j=1}^{F_N} \epsilon_j |j\rangle\langle j| + \sum_{j=1}^{F_N-1} T_j |j\rangle\langle j+1| \qquad \rho = T_A/T_B$$

$$H_N = \left(\frac{\rho}{2}H_{N-2} - T_B\right) \oplus \rho^2 H_{N-3} \oplus \left(\frac{\rho}{2}H_{N-2} + T_B\right)$$

Energy Spectrum



Spectrum's multifractal dimensions



$$d(q) = \lim_{l\to 0^+} \frac{1}{q-1}\frac{\log \sum_i \mu(K_i)^q}{\log l}$$

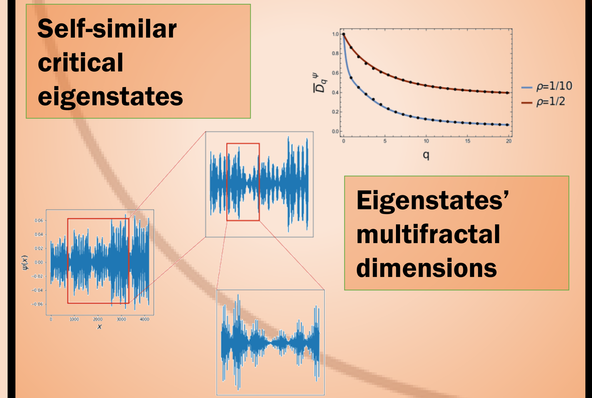
Fibonacci Substitution rule

$A \to AB,$

$B \to A,$

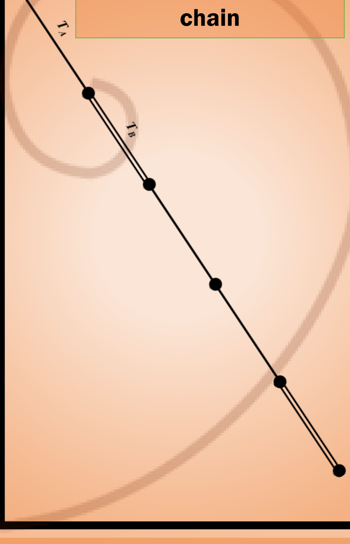
$\sigma^N(A) = ABAABABA\cdots \to |\sigma^N(A)| = F_N$

Self-similar critical eigenstates



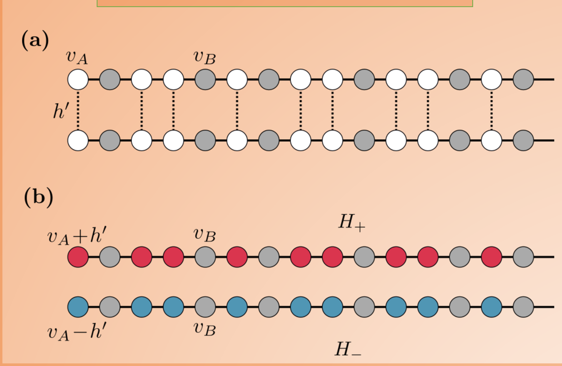
Eigenstates' multifractal dimensions

Section of a Fibonacci chain

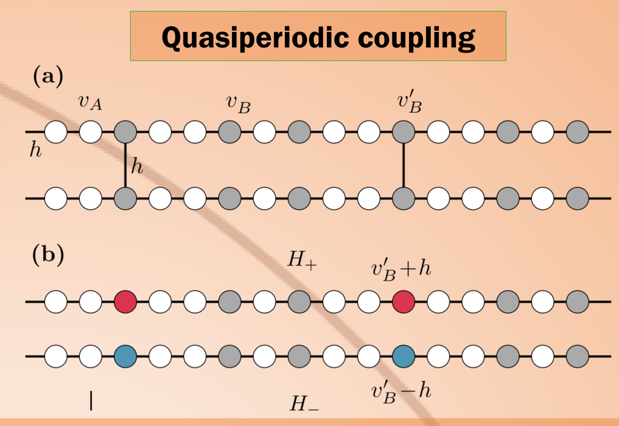


Different types of couplings

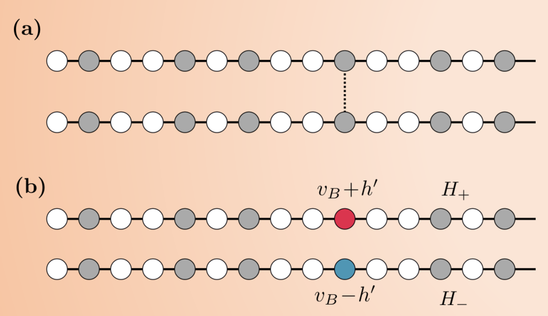
A (B)-site coupling



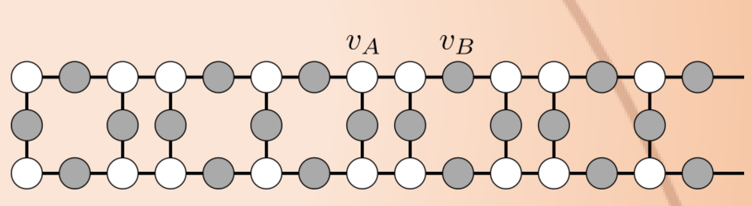
Quasiperiodic coupling



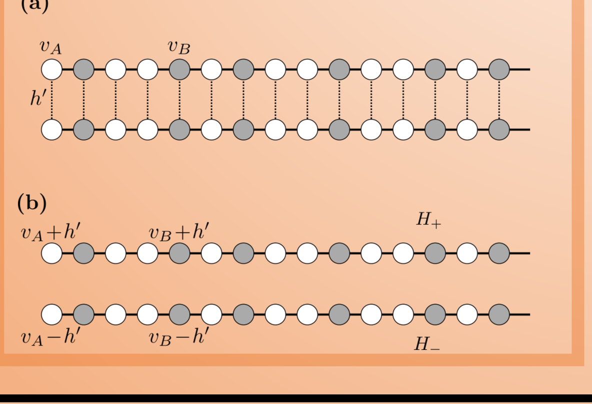
Single-site coupling

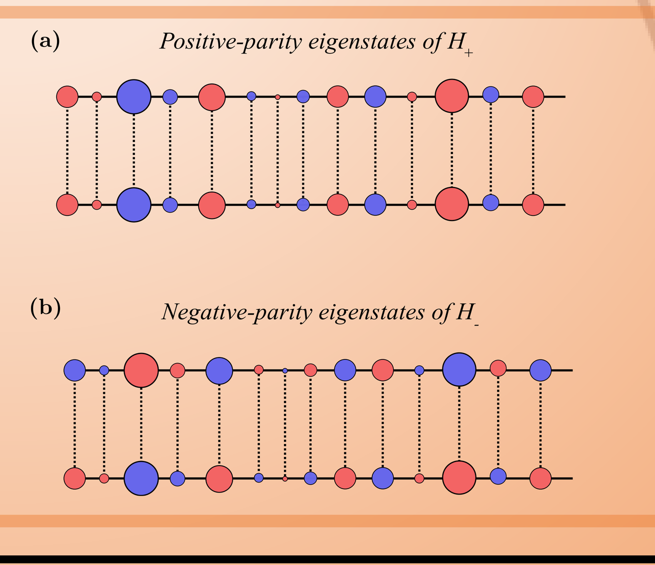


Intermediate-site coupling



Uniform coupling

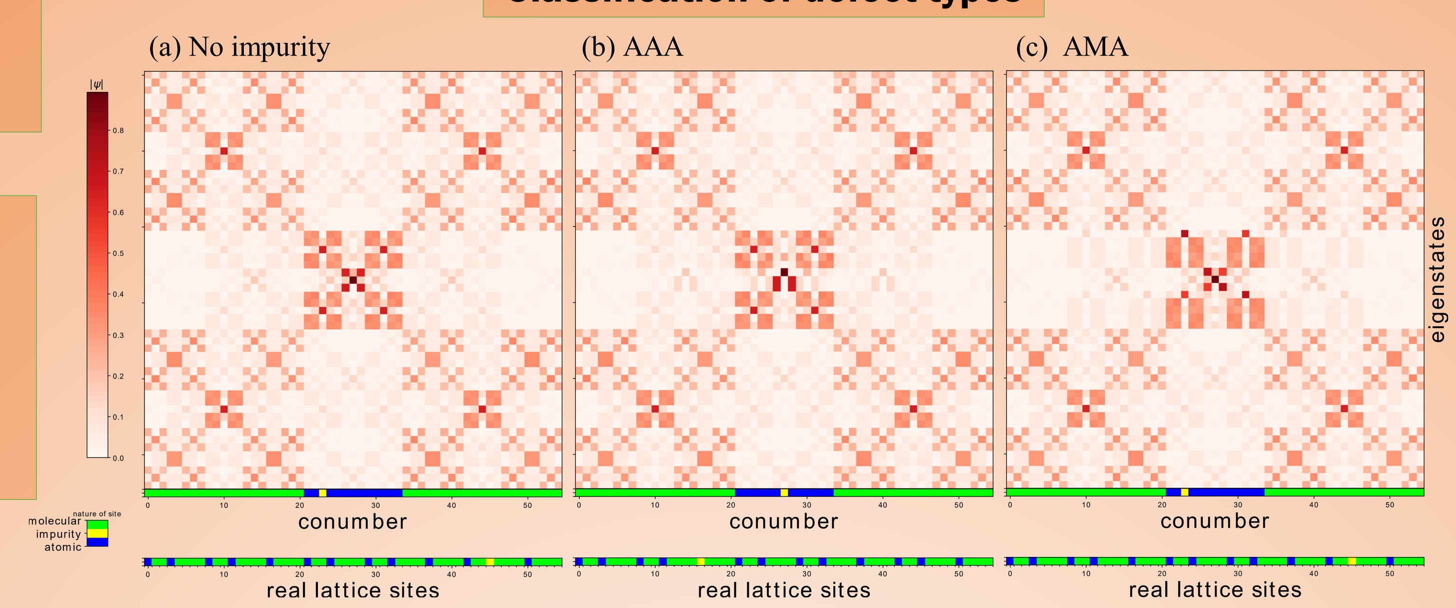




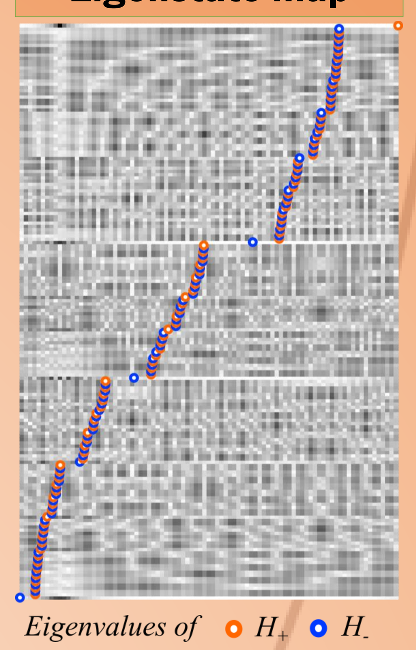
Single-site coupling

The single-site (defect) coupling only affects one particular cluster of eigenstates. See A.Moustaj et al. Phys. Rev. B 104, 144201 (2021).

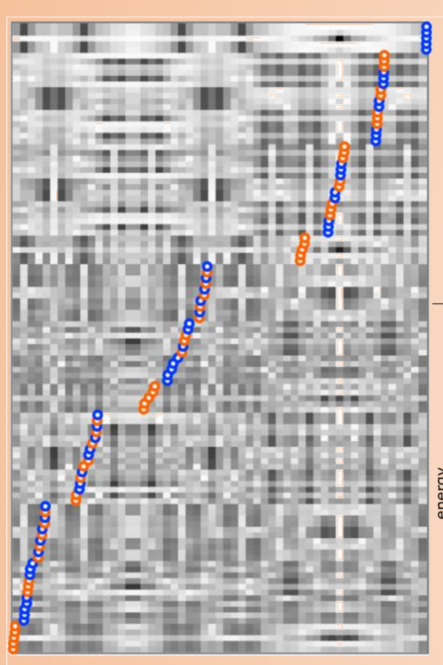
Classification of defect types



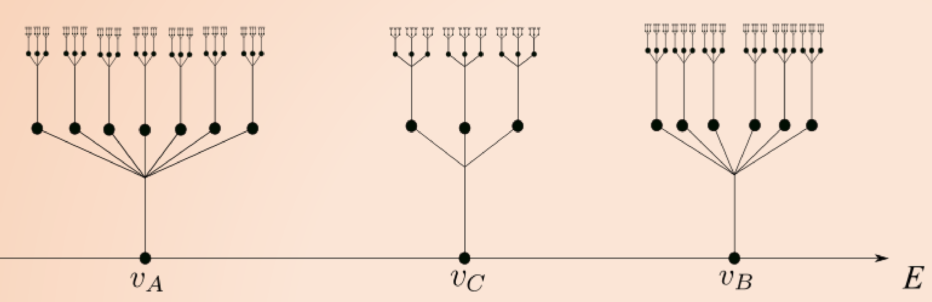
Eigenstate map

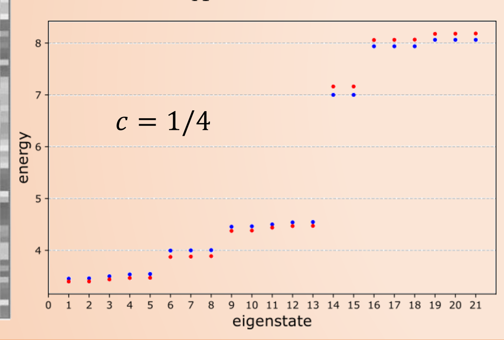


Quasiperiodic coupling



Hierarchical splitting from RG procedure



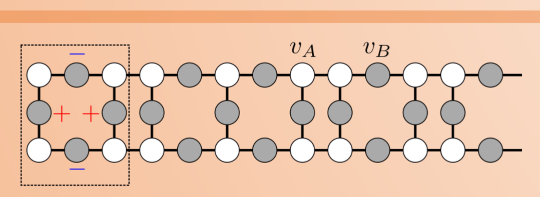


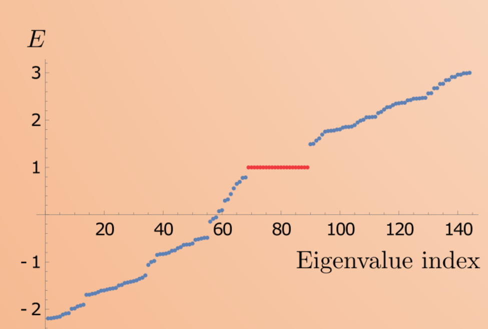
Comparison between theoretical calculation (blue) and numerical results (red) for contrast parameter value c=1/4

$$c = \left|\frac{h}{v_a - v_b}\right|$$

The eigenstates of the total system are still showing fractal behavior, albeit with a different multifractal curve. There is a slightly stronger preference for some states to localize, as can be observed from the eigenstate map.

Intermediate-site coupling

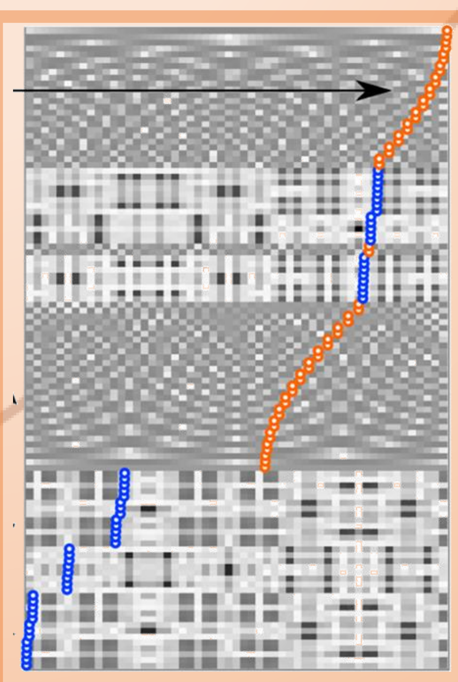




Compact localized states (CLS, dashed rectangle)

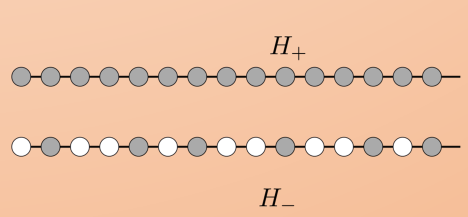
There are as many CLS's as there are plaquettes, and they are all eigenstates with the same energy eigenvalue. They generate a perfect flat band in the thermodynamic limit (see red dots).

A-site coupling



Coexistence of Bloch (orange) and critical eigenstates (blue).

Positive-parity eigenstates are extended Bloch waves (orange), while all negative-parity eigenstates are critical (blue). The spectrum is absolutely continuous for the Bloch sector, while it is singular continuous in the critical sector.



H+ is a periodic atomic chain, with unit lattice constant. H- is a Fibonacci chain, with shifted on-site potentials.

Conclusion:

Fibonacci ladders, with various inter-chain couplings, lead to numerous interesting physical situations! They exhibit a richer hierarchical energy structure, a coexistence of Bloch and critical eigenstates, and flat bands. Moreover, defects affect the system in an organized manner.