

Topological Phases of the interacting SSH model

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In collaboration with

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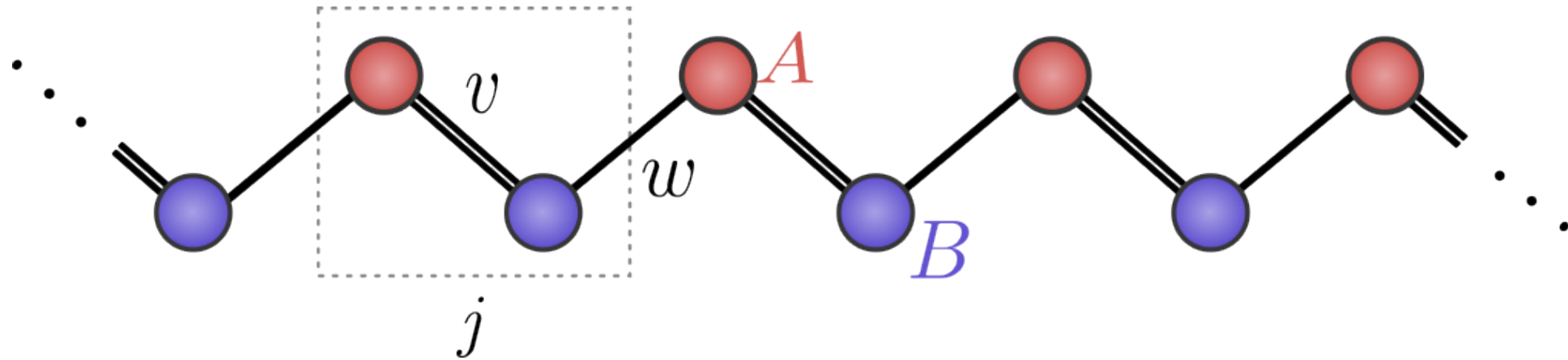


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SSH Model



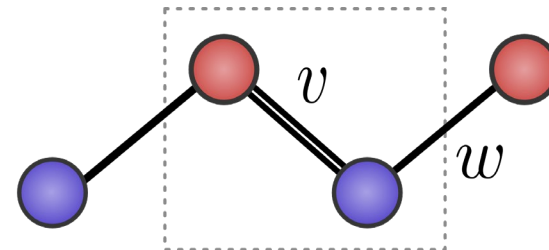
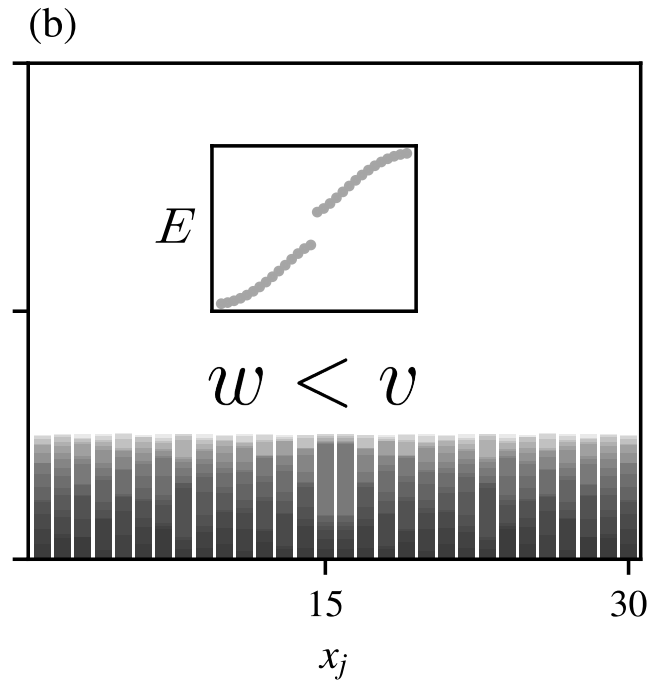
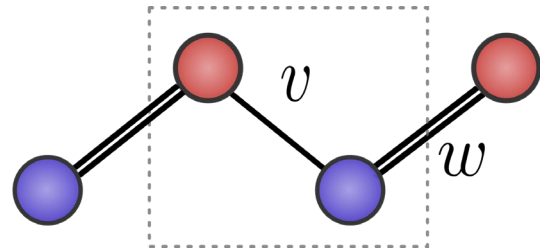
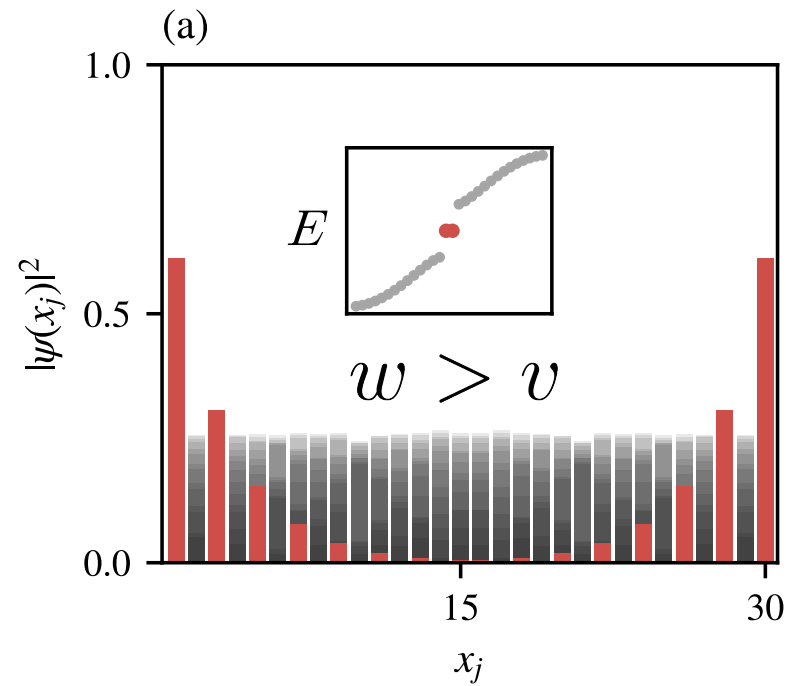
$$\hat{H}_0 = v \sum_j c_{j,A}^\dagger c_{j,B} + w \sum_j c_{j,B}^\dagger c_{j+1,A} + \text{h.c.}$$

Topology - Chiral symmetry

$$[H(k), \sigma_z]_+ = 0$$

$$\nu = \int_{-\pi}^{\pi} \frac{dk}{4\pi i} \text{Tr} \left[\sigma_z H^{-1}(k) \frac{dH(k)}{dk} \right] = \begin{cases} 1, & \text{if } w > v \\ 0, & \text{if } w < v \end{cases}$$

Bulk-Boundary Correspondence



Physical Observable - Polarization

$$P = e \langle \hat{X} \rangle$$

Modern Theory

Berry phase

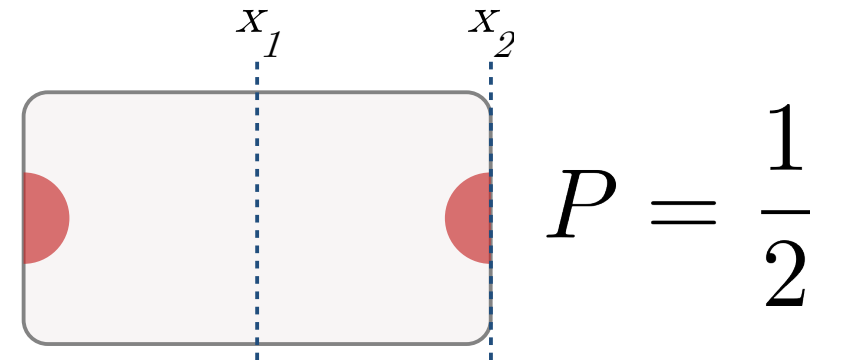
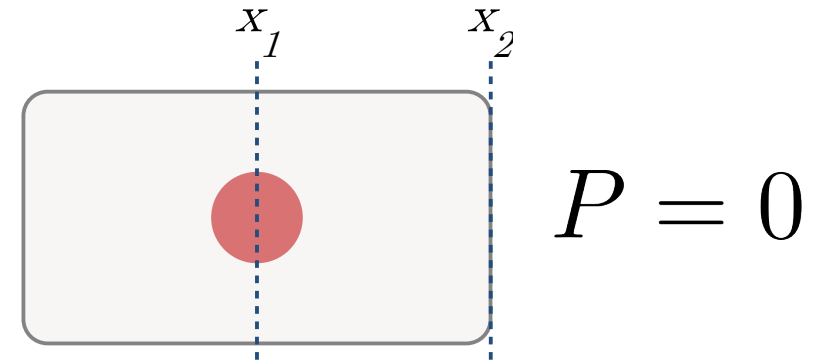


$$P = e \frac{\phi}{2\pi} \quad \text{mod } e$$

Polarization quantization

Inversion symmetry

$\phi = 0, \pi$ — $P = 0, \frac{1}{2}$



Polarization – topological response

$$P = \frac{1}{L} \frac{dF}{dE}$$

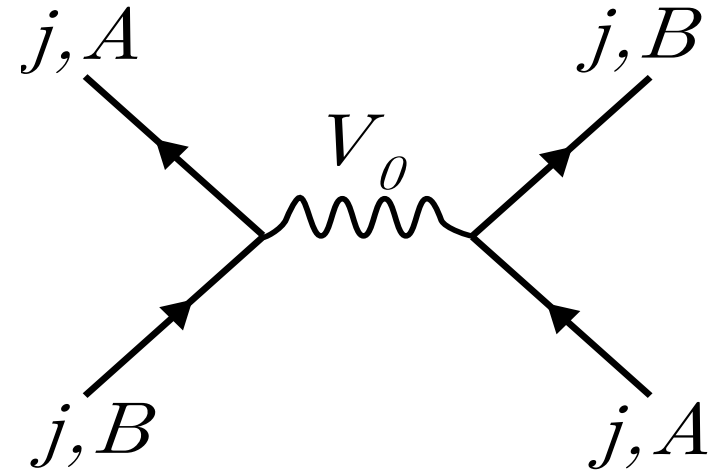
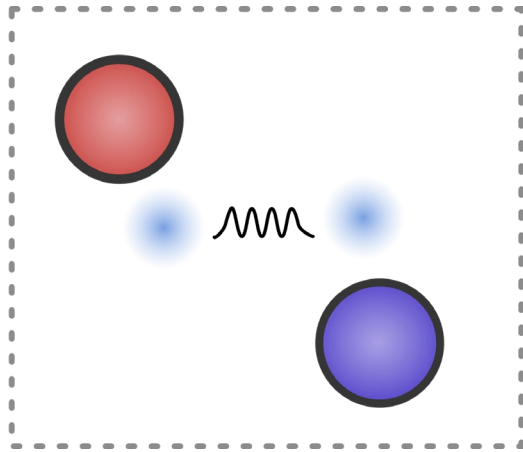


$$P = \int_{-\pi}^{\pi} \frac{dk}{4\pi i} \text{tr} \left[\sigma_z \mathbf{G}_0(k, 0) \partial_k \mathbf{G}_0^{-1}(k, 0) \right]$$



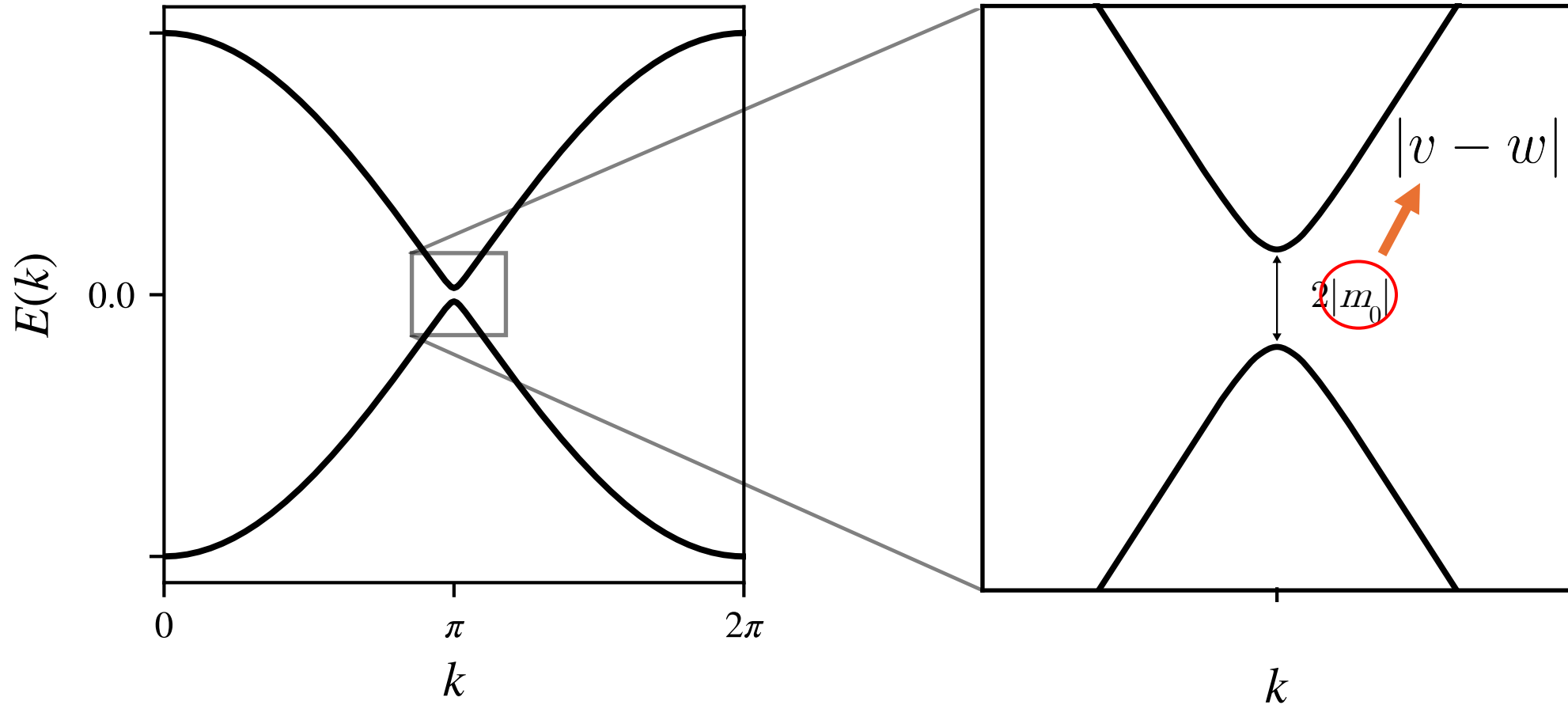
Green's function

Interactions - intracell



$$\hat{H}_{\text{int}} = V_0 \sum_j c_{j,A}^\dagger c_{j,A} c_{j,B}^\dagger c_{j,B}$$

Continuum limit – near $ka = \pi$



Continuum limit – Dirac Hamiltonian

$$\hat{H}_0 \longrightarrow \mathcal{H}_0 = \bar{\Psi}(x) (i v_F \gamma_1 \partial_x + m_0) \Psi(x)$$

wa

$$\hat{H}_{\text{int}} \longrightarrow \mathcal{H}_I = \frac{g}{2} [\bar{\Psi}(x) \gamma_\mu \Psi(x)]^2$$

$$V_0 a / 2$$

$$\gamma_0 = \sigma_x, \gamma_1 = i\sigma_z$$

Continuum limit – massive Thirring model

$$\mathcal{L} = \bar{\Psi}(x) (i v_F \not{\partial} - m_0) \Psi(x) - \frac{g}{2} [\bar{\Psi}(x) \gamma_\mu \Psi(x)]^2$$

Polarization – Continuum Limit $g = 0$

$$P = \int_{-\infty}^{\infty} \frac{dk}{4\pi i} \text{tr} \left[\sigma_z G_0(k, 0) \partial_k G_0^{-1}(k, 0) \right]$$

$$P = \frac{1}{2} \text{sign}(m_0)$$

Polarization – Continuum Limit

$$g \neq 0$$

$$P = \int_{-\infty}^{\infty} \frac{dk}{4\pi i} \text{tr} [\sigma_z G(k, 0) \partial_k G^{-1}(k, 0)]$$

$$P = \lim_{\Lambda \rightarrow \infty} [\varphi(\Lambda) - \varphi(-\Lambda)]$$



Phase of G.F.

Green's function – scaling behavior

$$G \sim |x|^{-2\Delta}$$



$$P = \pm\Delta$$

Scaling dimension of
the field in UV CFT

Green's function – scaling behavior

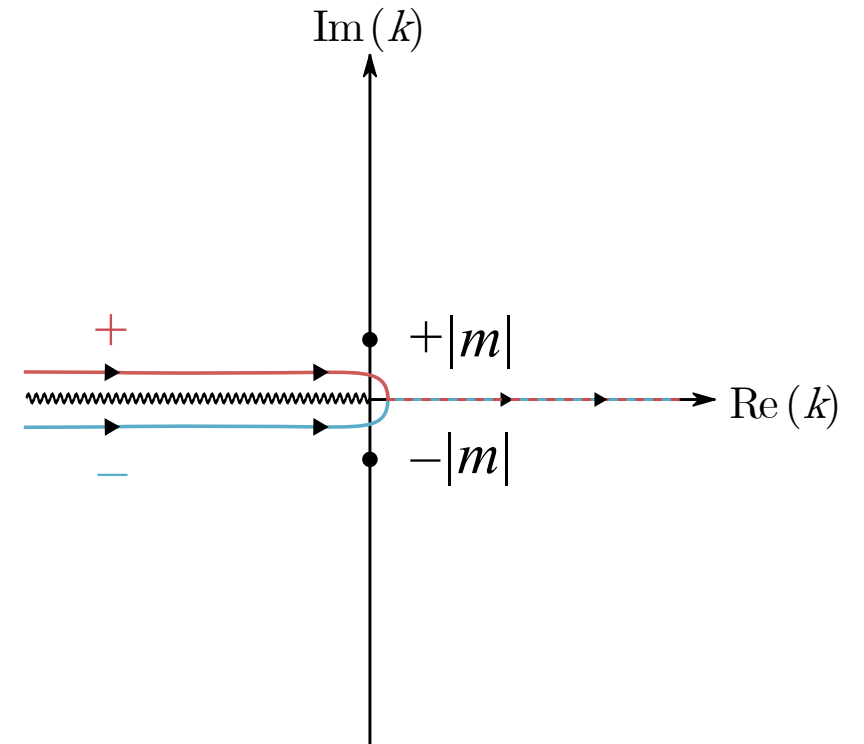
$$G \sim |x|^{-2\Delta}$$



$$P = \pm \Delta$$



Depends on path of integration!



Green's function – scaling behavior

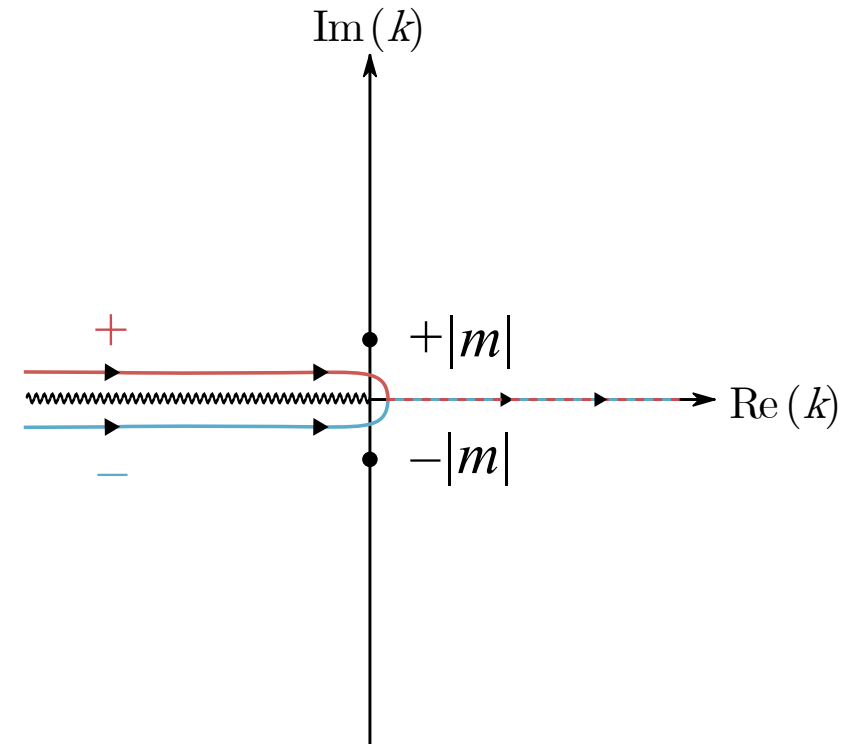
$$G \sim |x|^{-2\Delta}$$



$$P = \text{sign}(m) \Delta$$



Protection provided by
IR mass scale



Green's function – scaling behavior

$$G \sim |x|^{-2\Delta}$$

$$P = \text{sign}(m)\Delta$$

Scaling dimension of
the field in UV CFT

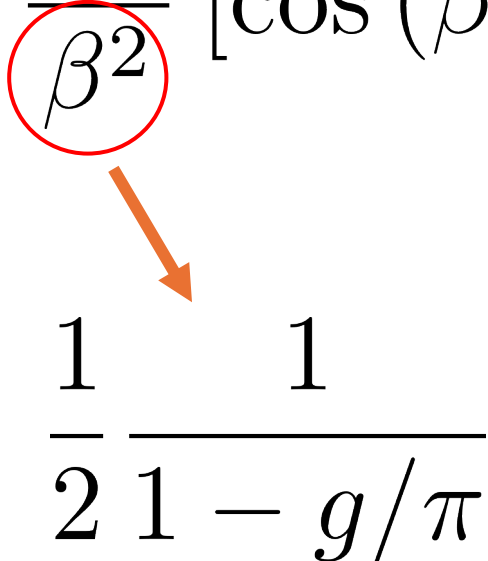
How to determine this?

Duality transformation

$$\Psi(x, t) = \exp \left\{ \frac{i}{2} \left[\beta \Phi(x, t) + \frac{1}{\beta} \Theta(x, t) \right] \right\}$$
$$\bar{\Psi}(x, t) = \exp \left\{ -\frac{i}{2} \left[\beta \Phi(x, t) - \frac{1}{\beta} \Theta(x, t) \right] \right\}$$

Bosonic Fields

Sine-Gordon model

$$\mathcal{L}_{\text{SG}} = \frac{1}{2} (\partial_\mu \Phi)^2 - \frac{m_0^2}{\beta^2} [\cos(\beta\Phi) - 1]$$

$$\frac{1}{2} \frac{1}{1 - g/\pi}$$

Green's function – form factors

$$G(k) = \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} \frac{dx}{2\pi} \frac{d\theta_1}{2\pi} \cdots \frac{d\theta_n}{2\pi} e^{ikx} |F_n^{\tilde{O}}(\theta_1, \dots, \theta_n)|^2$$
$$\times \delta \left(k - m \sum_{j=1}^n \sinh \theta_j \right).$$

Exact form factors in UV region yield

$$\Delta = \frac{1}{4} \left(\frac{1}{1 - g/\pi} + 1 - \frac{g}{\pi} \right)$$

Polarization of interacting model

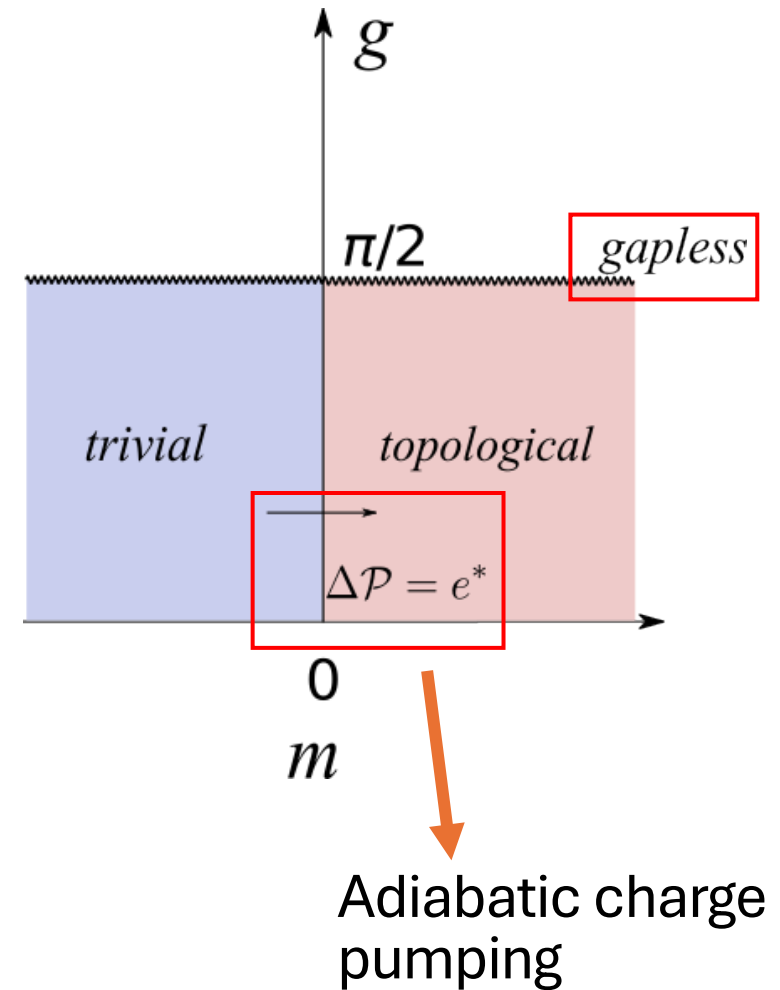
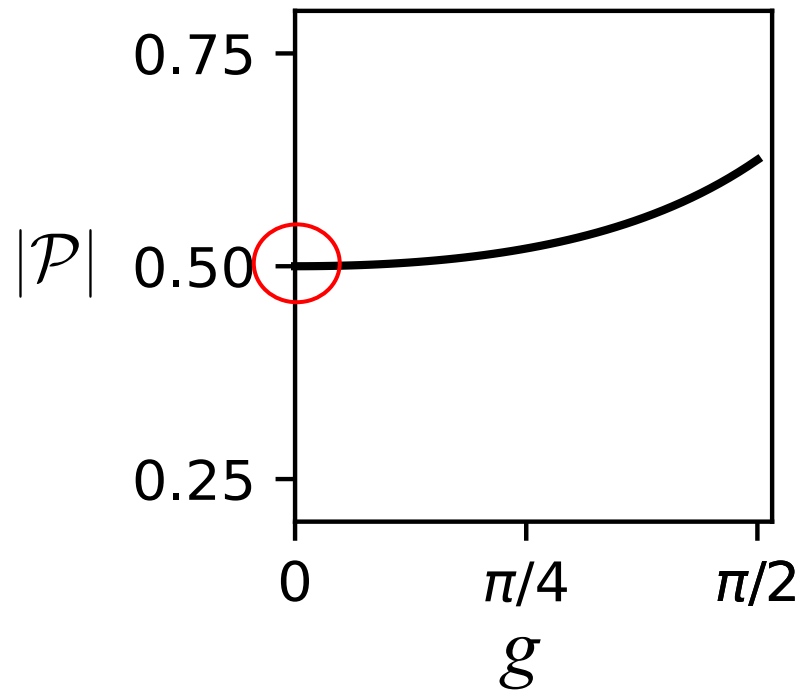
$$P = \frac{1}{4} \left(\frac{1}{1 - g/\pi} + 1 - \frac{g}{\pi} \right) \text{sign}(m_0)$$

String operator contribution

Renormalization of fermionic charge

Protection provided by IR mass scale

Polarization of interacting model



Further Insights – RG

$$\left[\Lambda \frac{\partial}{\partial \Lambda} + \sum_i \beta_{\text{RG}}(g_i) \frac{\partial}{\partial g_i} + n\gamma \right] G^{(n)}(k_1, \dots, k_n; \Lambda, g) = 0$$

Scaling dimension of field

$$\beta_{\text{RG}} \rightarrow 0 \quad \longrightarrow \quad \text{Im log } G^{(2)} \sim 2\gamma \text{Im log}(k/\Lambda)$$

Take-home message

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- Study interacting SSH using polarization
- Continuum limit to obtain Thirring model
- Exact results provide insights on topological phase
- Topological phase protected by IR (mass)
- Magnitude of Polarization determined by UV (scaling dimension)

Team



Labor force



Supervisors

