Emergent Non-Hermitian Models

Phys. Rev. B **109**, 045122 (2024)

Presenter: Anouar Moustaj

Supervisor: Cristiane Morais Smith

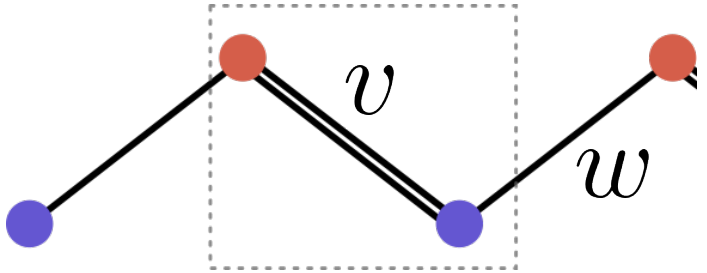
In collaboration with

Lumen Eek, Malte Rontgen,

Vassos Achilleos, Vincent Pagneux

Paradigmatic Models in 1D

- Useful for studying specific aspects
- Allow analytic expressions
- Can understand behaviour without complications



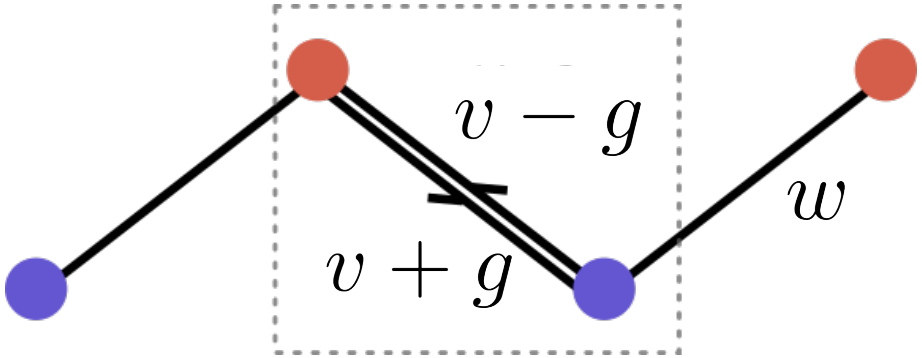
Phys. Rev. Lett. **42**, 1698 (1979)

SSH Chain – Non Hermitian

- Complex Energies

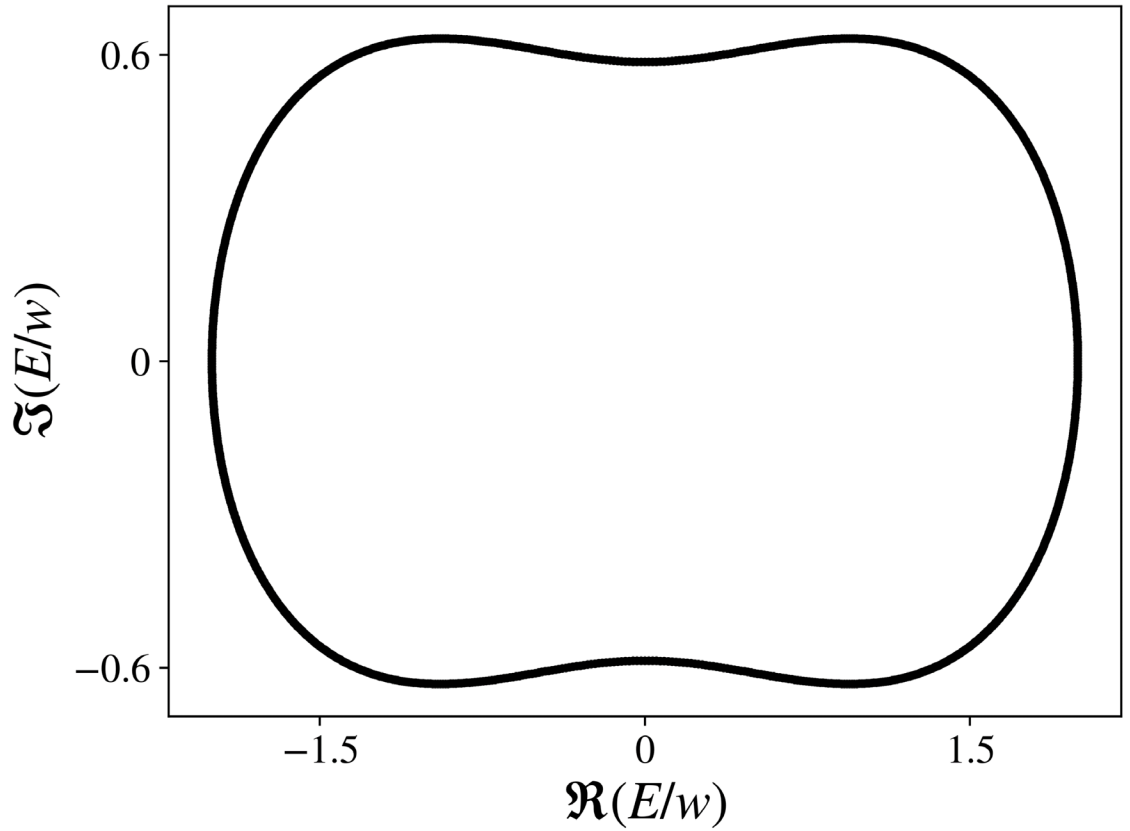
$$H \neq H^{\dagger}$$

- Enriched Band Topology and more Symmetry Classes
- Enriched topological classification
- Unusual boundary phenomena

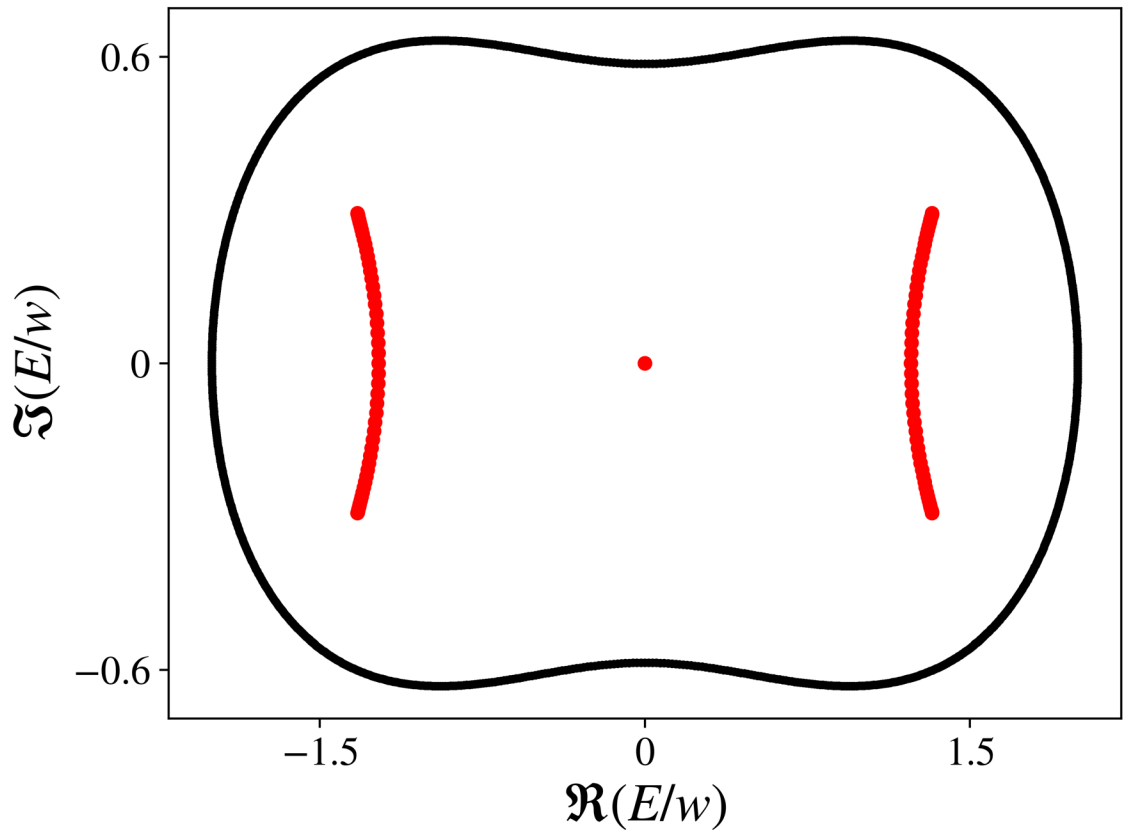


ACS Photonics, **5,** 6, 2271–2279 (2018)
Phys. Rev. B **97**, 045106 (2018)

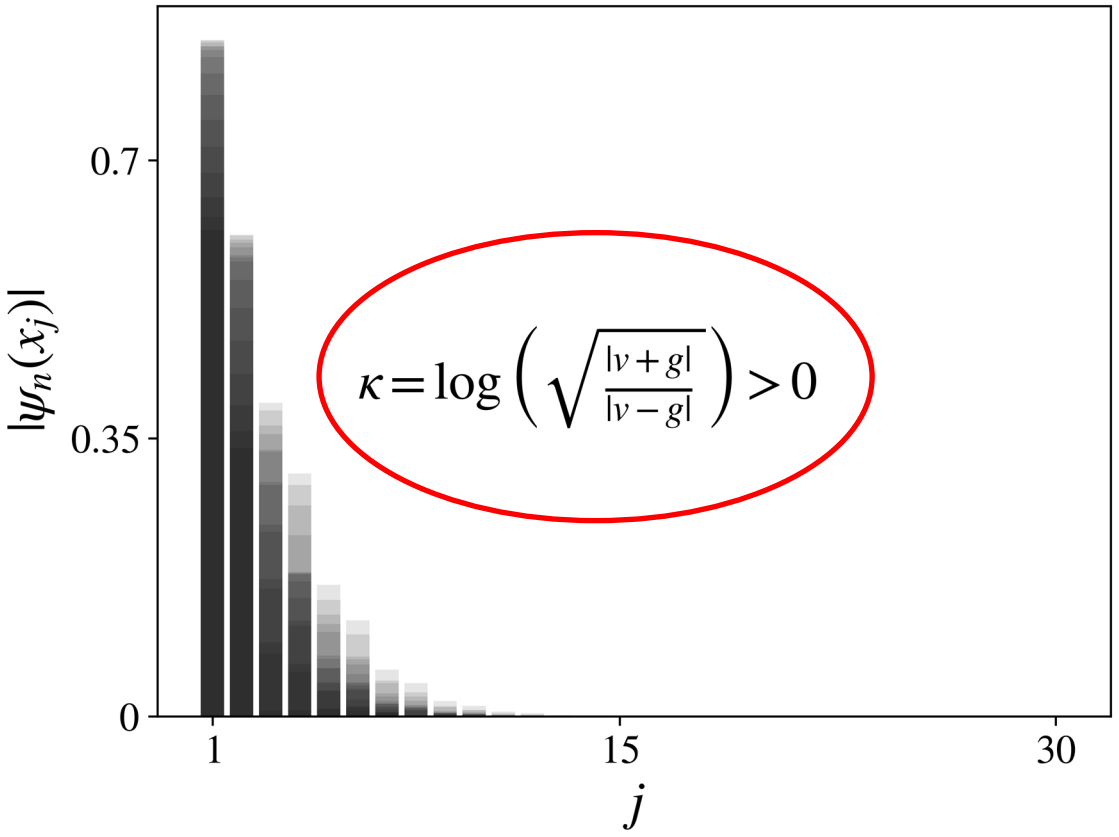
SSH Chain – Non Hermitian: PBC $\neq$ OBC

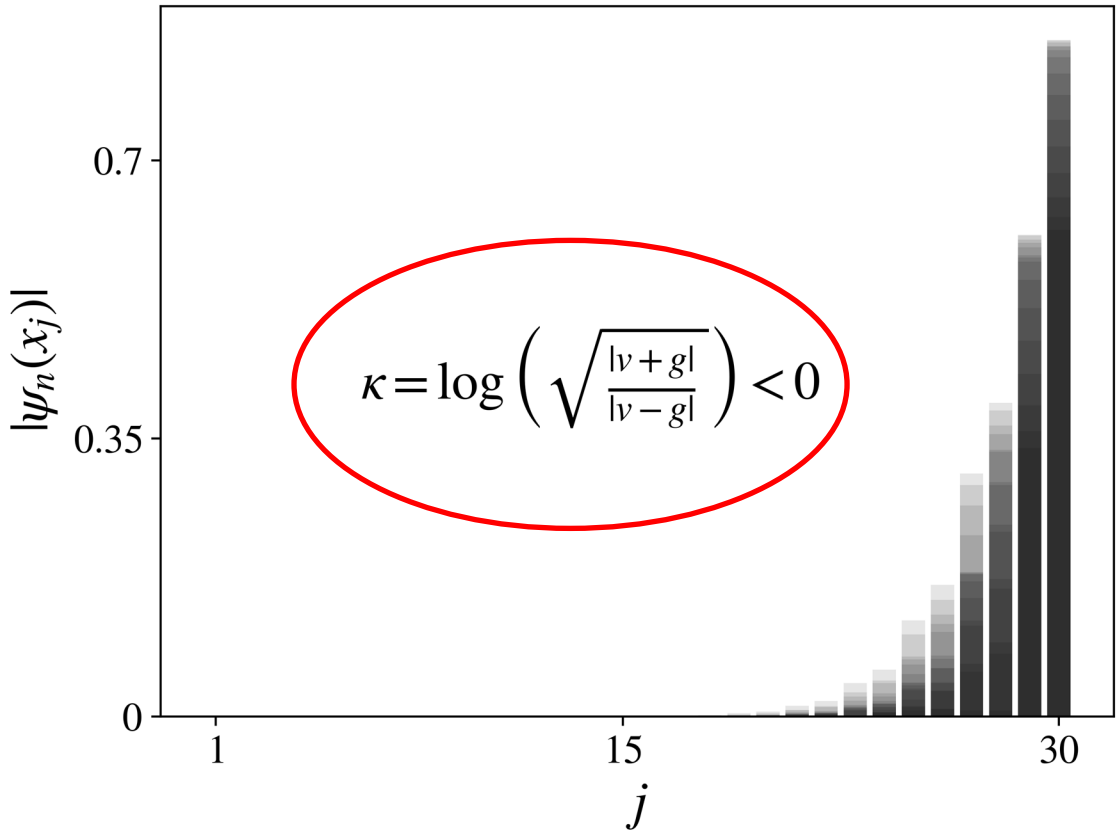


SSH Chain – Non Hermitian: PBC $\neq$ OBC



SSH Chain – Skin Effect





Phys. Rev. Lett. **121**, 086803 (2018)

SSH Chain – Topology: Chiral Symmetry

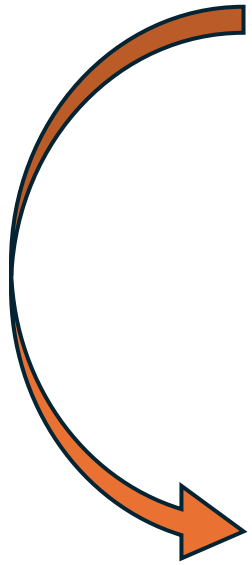
$$[H, S]_+ = 0 \quad \Rightarrow \quad S = \sigma_z$$

$$\nu = \int_{-\pi}^{\pi} \frac{dk}{4\pi i} \mathrm{Tr}\left[S H^{-1}(k) \frac{dH(k)}{dk} \right] = \begin{cases} 1, & \text{if } w > v \\ 0, & \text{if } w < v \end{cases}$$

$$\pi_1(S^1) = \mathbb{Z}$$

Rev. Mod. Phys. **88**, 035005 (2016)

SSH Chain – Non-Hermitian Topology

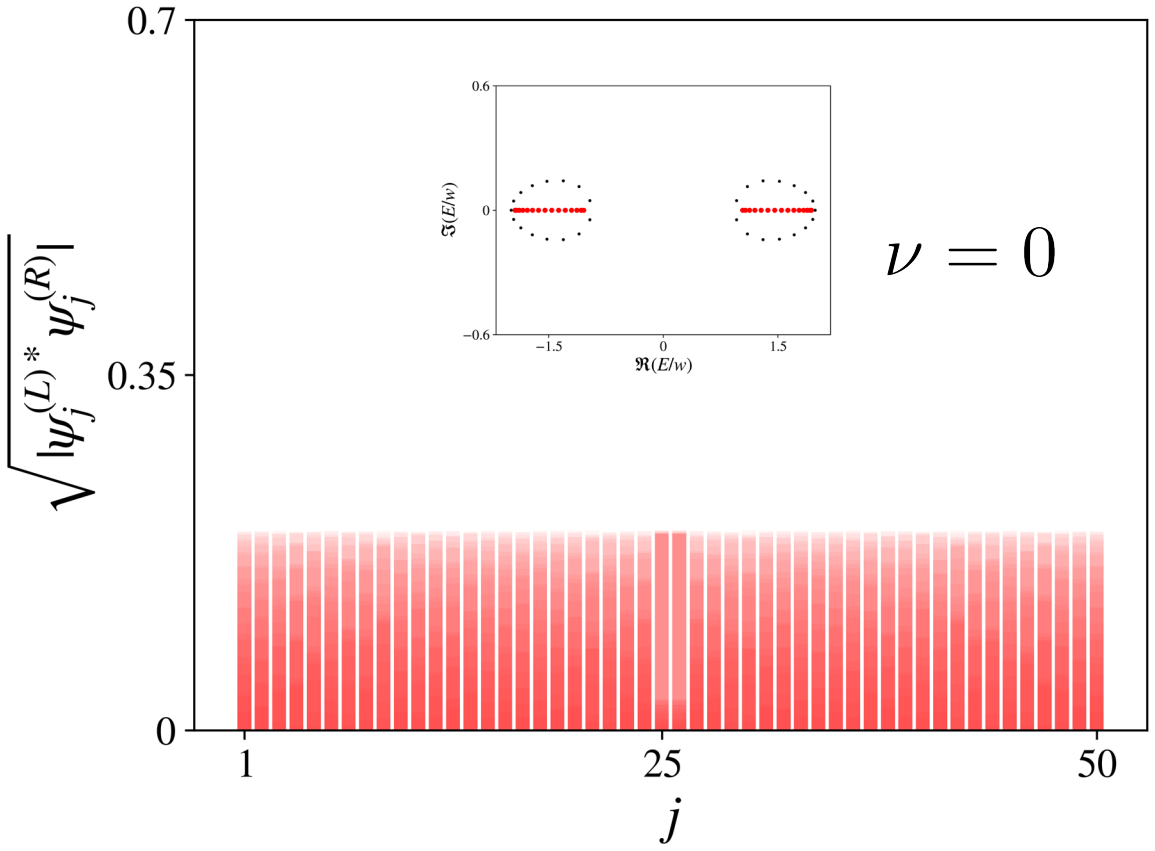


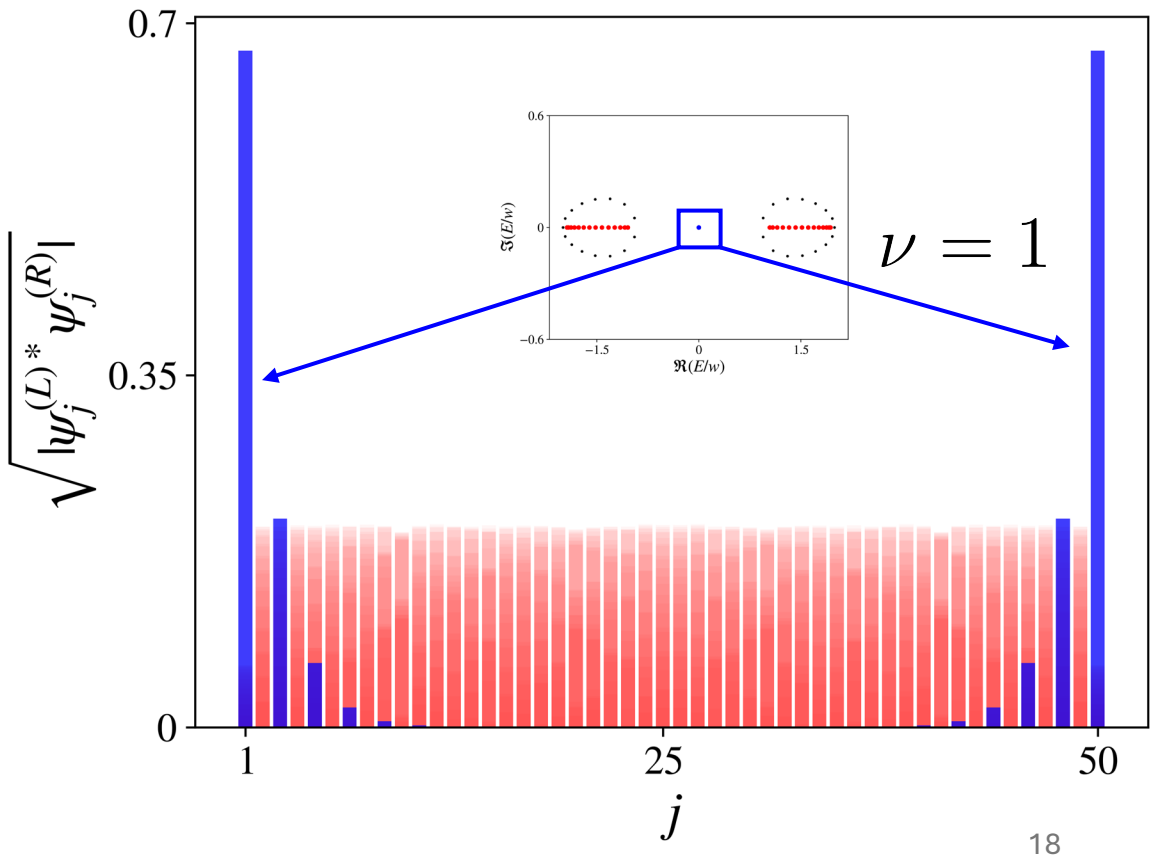
$$\nu = \begin{cases} 1, & \text{if } w > v \\ 0, & \text{if } w < v \end{cases}$$

$$\nu = \begin{cases} 1, & \text{if } |w| > \sqrt{|v^2 - g^2|} \\ 0, & \text{if } |w| < \sqrt{|v^2 - g^2|} \end{cases}$$

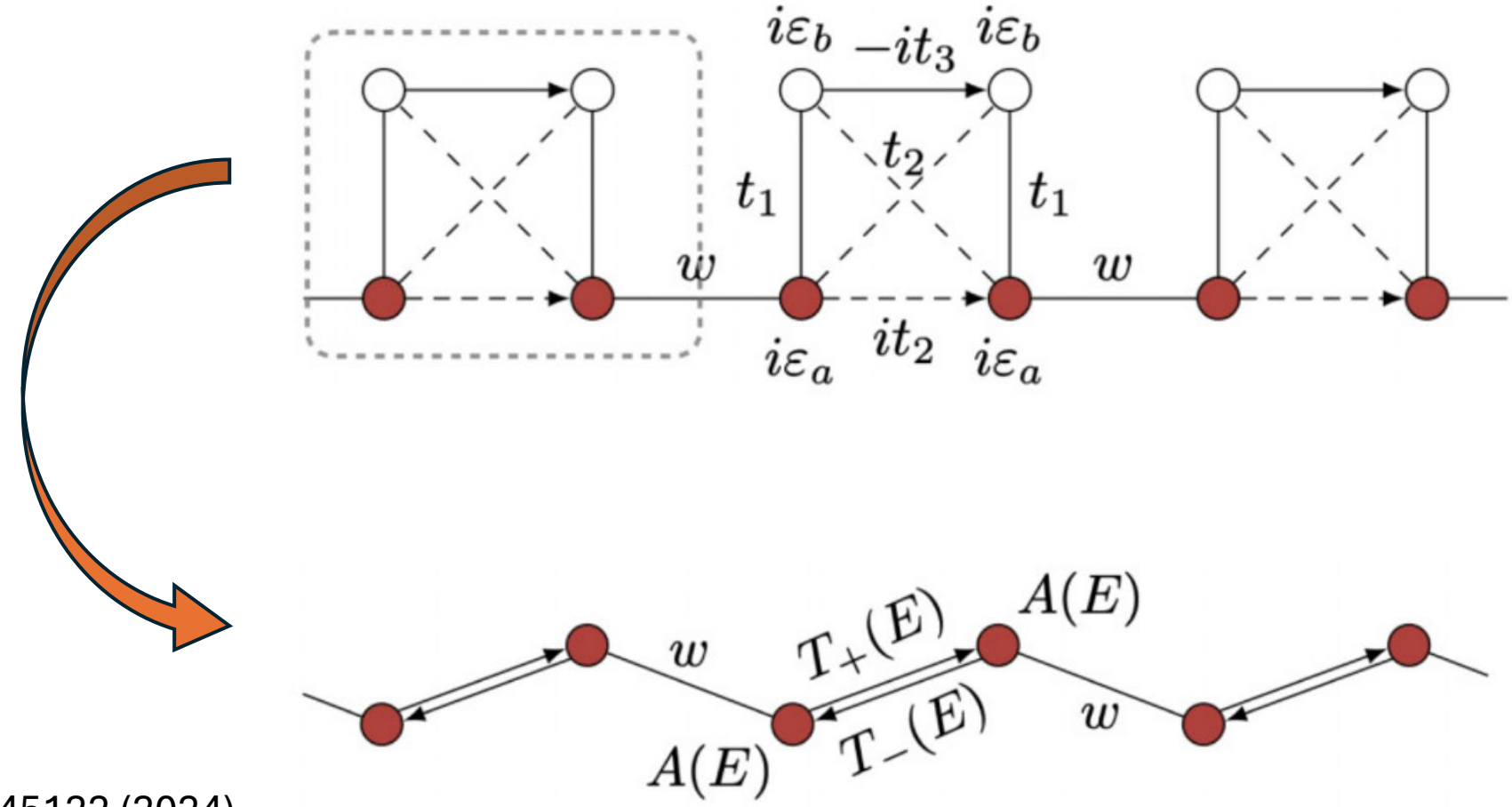
Phys. Rev. Lett. **121**, 086803 (2018)

SSH Chain – Non-Hermitian Topology





Complicated Unit Cells – Hidden Topology?



Phys. Rev. B **109**, 045122 (2024)

Isospectral Reduction

$$H\left|\psi\right\rangle = E\left|\psi\right\rangle \qquad H \equiv \begin{pmatrix} H_{SS} & H_{S\overline{S}} \\ H_{\overline{S}S} & H_{\overline{S}\overline{S}} \end{pmatrix}$$

ISR

$$\mathcal{H}_S(E) = H_{SS} - H_{S\overline{S}}\left(H_{\overline{S}\overline{S}} - E\mathbb{1}\right)^{-1} H_{\overline{S}S}$$

Phys. Rev. B **109**, 045122 (2024)

Isospectral Reduction – NH SSH

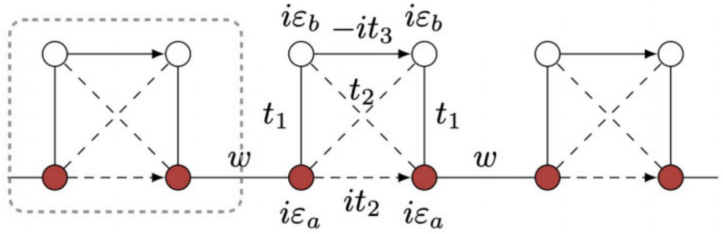
$$H_R(k,E) = \begin{pmatrix} A(E) & T_+(E) + we^{-ik} \\ T_-(E) + we^{ik} & A(E) \end{pmatrix}$$

Latent Chiral Symmetry Condition: $A(E_t) - E_t = 0$

$\tilde{H}(k) \equiv H(k, E_t) - E_t \mathbb{1}_{2\times 2}$ → Calculate invariant! $\nu(E_t)$

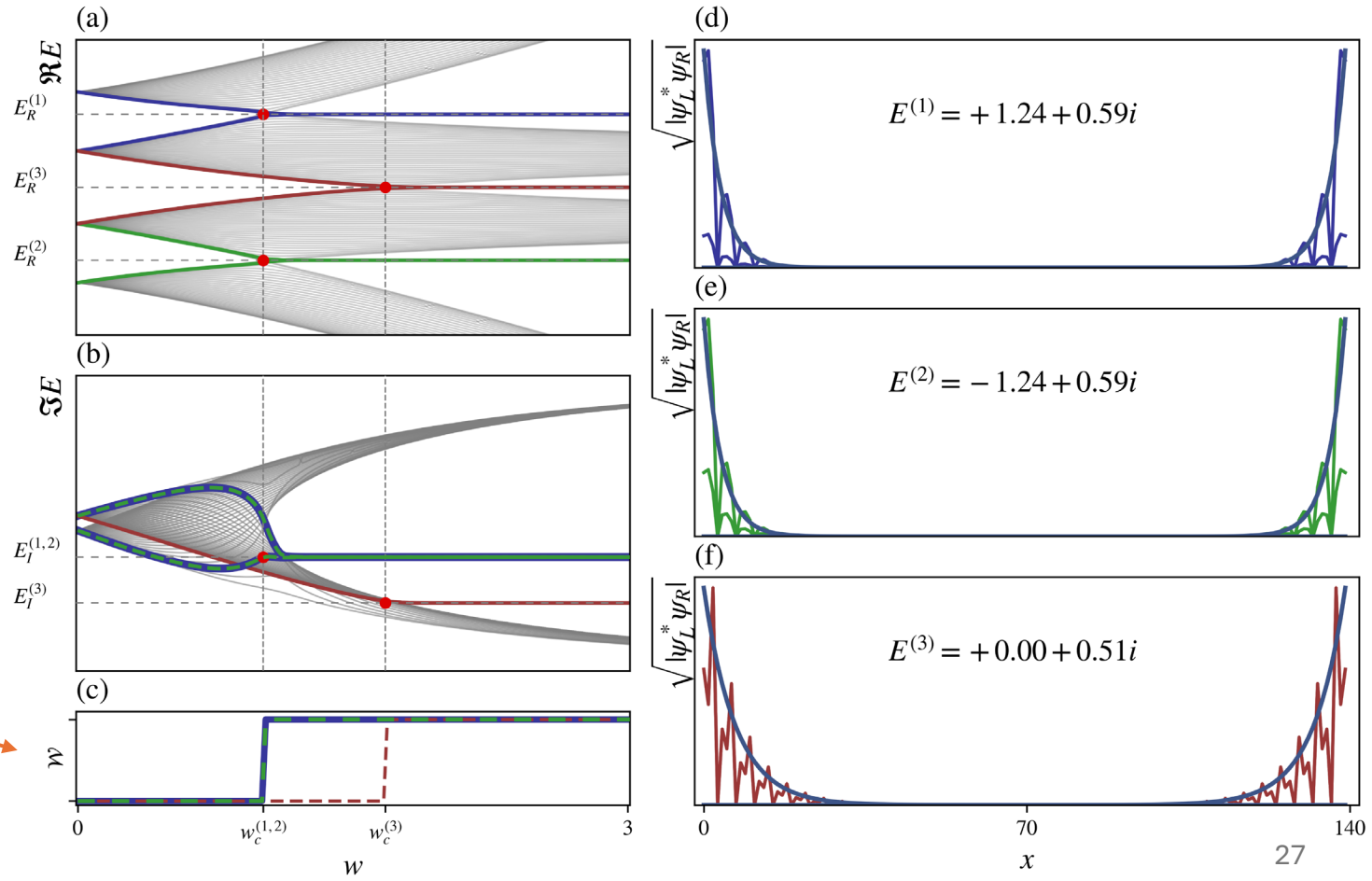
Phys. Rev. B **109**, 045122 (2024)

Multiple Edge Modes



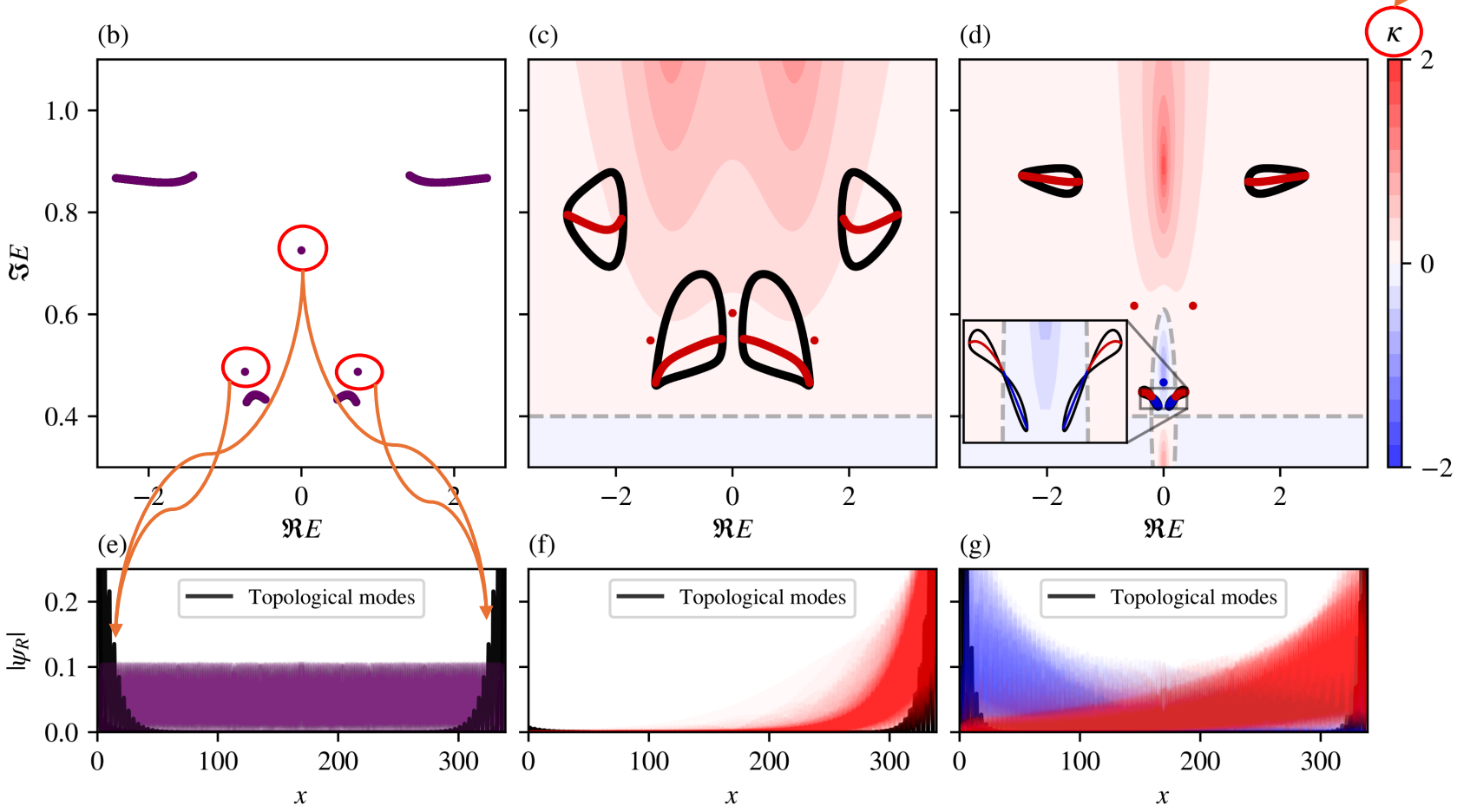
$$\nu(E_t) = \begin{cases} 1, & \text{if } |w| > |\alpha(E_t)| \\ 0, & \text{if } |w| < |\alpha(E_t)| \end{cases}$$

$$|\alpha| = \sqrt{|v^2(E_t) - g^2(E_t)|}$$



Phys. Rev. B **109**, 045122 (2024)

Possibility of Double-Sided Skin Effect

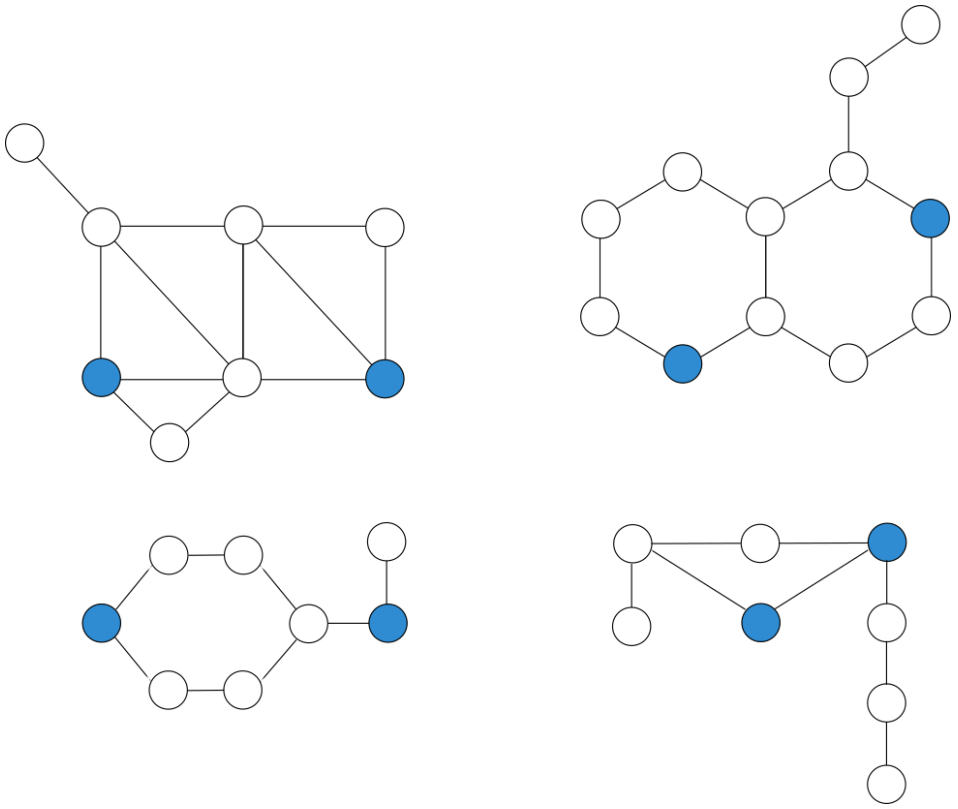


$$\kappa(E) = \log\left(\sqrt{\left|\frac{v(E)-g(E)}{v(E)+g(E)}\right|}\right)$$

$$\xi_L(E) = \frac{1}{\log\left|\frac{v(E)-g(E)}{w}\right|},$$

$$\xi_R(E) = \frac{1}{\log\left|\frac{v(E)+g(E)}{w}\right|},$$

General Construction Principles



Blue Sites are Latently Symmetric

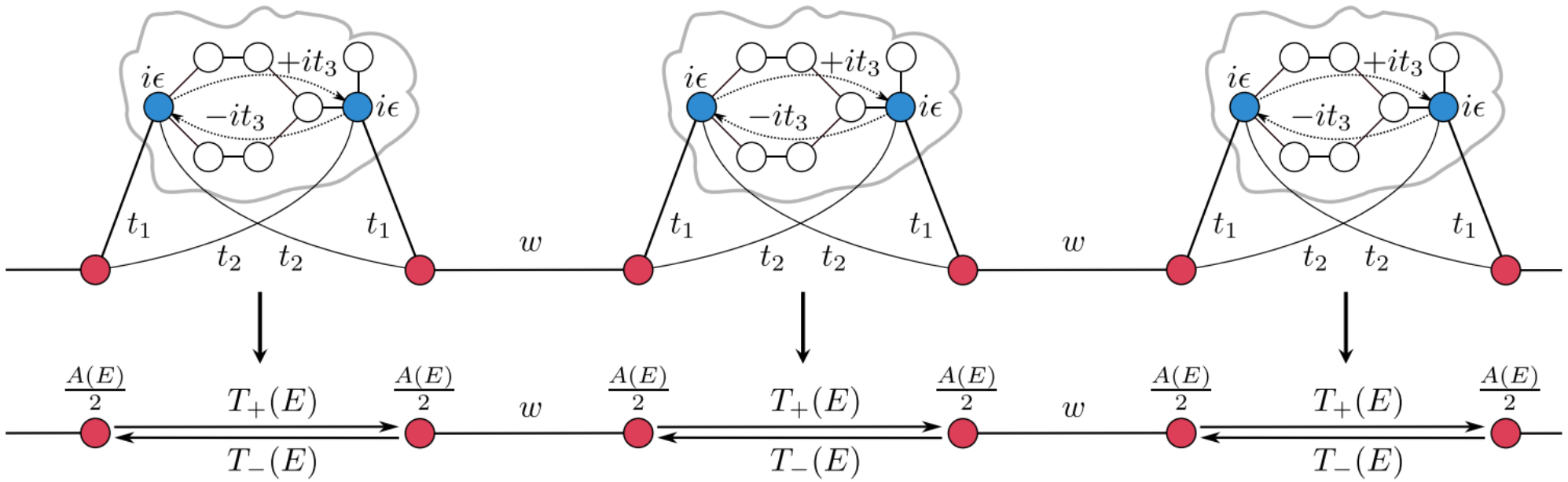
$$\mathcal{R}_S(G,E) = \begin{pmatrix} A(E) & B(E) \\ B(E) & A(E) \end{pmatrix}$$

$$[\mathcal{R}_S(G,E), \mathcal{M}]_- = 0$$

$$\mathcal{M} = \sigma_x$$

Phys. Rev. B **109**, 045122 (2024)

General Construction Principles



Phys. Rev. B **109**, 045122 (2024)

Conclusion and Outlook

- Paradigmatic models are useful for studying specific aspects
- We reduced complicated models to energy dependent simple models
- We predict properties of the full model based on energy dependent properties of the simple models: topology and skin-effect in NH systems
- Outlook: Apply to more paradigmatic models and other latent symmetries
- Read More: Phys. Rev. B **109**, 045122 (2024)